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c) $f(x) = u(x)(v(x)^{-1})$ Let $u = u(x)$ and $v = \frac{1}{v(x)}$

$f'(x) = u'v + uv'$

$= \frac{u'(x)}{v(x)} + \frac{u(x)v'(x)}{(v(x))^2}$ $\quad v' = -\frac{1}{(v(x))^2}\frac{dv}{dx}$

$= \frac{u'(x)v(x) + u(x)v'(x)}{(v(x))^2}$

Do We Need the Quotient Rule?

The quotient rule can be difficult to memorize, and some students are more comfortable with negative exponents than they are with fractions. In this exercise we learn how we can use the chain and product rules together in place of the quotient rule.

a) Use the quotient rule to find the derivative of $\frac{x^3}{x+1}$.

b) Use the product and chain rules to find the derivative of $x^3 \cdot (x+1)^{-1}$. Note that $x^3 \cdot (x+1)^{-1} = \frac{x^3}{x+1}$.

c) Use the chain and product rules (and not the quotient rule) to show that the derivative of $u(x)(v(x))^{-1}$ equals $\frac{u'(x)v(x) - u(x)v'(x)}{(v(x))^2}$.

a) $y = \frac{x^3}{x+1}$

$\frac{dy}{dx} = \frac{3x^2(x+1) - x^3(1)}{(x+1)^2}$

$= \frac{3x^3 + 3x^2 - x^3}{(x+1)^2}$

$= \frac{2x^3 + 3x^2}{(x+1)^2}$

$= \frac{x^2(2x+3)}{(x+1)^2}$

b) $\frac{dy}{dx} = 3x^2(x+1)^{-1} + (-(x+1)^{-2})(1)(x^3)$

$= \frac{3x^2}{x+1} - \frac{x^3}{(x+1)^2}$

$= \frac{3x^2(x+1) - x^3}{(x+1)^2}$

$= \frac{2x^3 + 3x^2}{(x+1)^2} = \frac{x^2(2x+3)}{(x+1)^2}$

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